Fermi National Accelerator Laboratory

FN-515

# Space Charge, Synchrotron Oscillations and Multiparticle Tracking

S. R. Mane
Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510

May 12, 1989

Operated by Universities Research Association, Inc., under contract with the United States Department of Energy

# Space Charge, Synchrotron Oscillations and Multiparticle Tracking

S.R. Mane
Fermi National Accelerator Laboratory, Batavia, Illinois 60510

May 12, 1989

## 1 Introduction

The broad goal of the project described by the title is to track particles through not only a set of magnetic fields, but also to include their mutual repulsion, the so-called space charge repulsion. The motivation for this study comes from the behavior of the Fermilab Main Ring at injection (8 GeV), where it is speculated that space charge may be responsible for significant particle loss. Fig. 1 contains an example of particle losses from the Main Ring [1]. It is observed that most of the losses are at low energy, and at transition, while losses are negligible after transition. Since the space charge force increases as the energy decreases, it is speculated that the observed losses are induced by space charge. The motivation for the work described below comes entirely from this correlation. The plan, therefore, is to employ particle tracking, with and without space charge, to follow particles around a Main Ring lattice, to see if one can quantify the effects (presumed deleterious) of space charge. Numerical particle tracking is the chosen technique because analytical calculations, for a many-body system in a set of numerous nonlinear magnetic multipoles, with random magnet misalignments, are too difficult.

The long-term plan is to obtain graphs of beam behavior similar to those observed experimentally in the Fermilab Main Control Room, e.g. Fig. 1. This is a very ambitious task, and requires particle tracking using a realistic lattice model, as well as full six-dimensional tracking, i.e. inclusion of both longitudinal and transverse degrees of freedom. This report describes some steps in the above direction. The reader should be warned at once that this report will fall far short of the above final goal. It will merely contain (i) a description of a multiparticle tracking program, specially written to include space charge and synchrotron oscillations, but treating only an idealized lattice (to be described below), and (ii) particle tracking using a Main Ring lattice [2] using the program TEVLAT [3]. A version of this program was used to study space charge effects in the Main Ring in a previous report [4]. Since then, TEVLAT has been modified, and it has also been interfaced with the interactive graphics program AESOP by L. Michelotti [5].

## 2 Fundamental Problems

There are several problems about the foundations of this entire project, not just about cosmetic details at the end. Although the general concept of particle tracking is clear, it is *not* clear how to process the resulting information. The "data analysis" depends very much on the specific problem being studied. However:

1. TEVLAT produces different answers on different computers, for the same lattice model and input parameters,
2. the tracking is slow, especially with space charge, and it takes a long time to track for several thousand turns,
3. no method appears to be available, in six dimensions, to predict the long-term behavior of a trajectory from its short-term behavior (here "long-term" means a few seconds, i.e. a few million turns, while "short-term" is not well defined),
4. there is no clear-cut method to determine the dynamic aperture of the Main Ring from the tracking data (there are arguments about the precise definition of the dynamic aperture),
5. one does not know all the relevant effects that must be included; typically the importance of an effect is determined from its influence on the tracking results — *a posteriori*, not *a fortiori*,
6. one must perform a Monte Carlo analysis to obtain a reasonable estimate of the effects of the random multipoles in the ring; this requires a model of the randomness of the multipoles, e.g. correlations between magnet misalignments,
7. TEVLAT tracks only one particle, not a distribution; extensive modifications will be required to change this,
8. it is not clear how to relate the final results to the TEVLAT input parameters, i.e. to claim that one "understands" the results,
9. beyond simple results such as the Laslett tuneshift [6], there are no checks that the results are correct when space-charge is included.

One sees therefore that there are serious problems with tracking in general, and with the use of TEVLAT. However, Michelotti and coworkers are taking active steps to study particle tracking [7], although without space charge so far. Efforts are being made to characterize regular and chaotic orbits, to develop methods for predicting the long-term behavior of an orbit, not only for simple models but also for data using detailed Main Ring lattices. Work on particle tracking is also being actively pursued at CERN and SSC/CDG [8]. Most of these results are in two or four dimensions, and treat only transverse particle motion. The particles are independent; no collective effects are treated. There appeared to be no consensus in Ref. [8] as to the definition of the dynamic aperture, nor a criterion to quantify the tolerances on magnetic field quality, etc. No results in this report were presented in Ref. [8].

## 3 TEVLAT

The program TEVLAT [3] is a kick code, and tracks one particle around a given ring lattice. In its present form, it can treat arbitrary lattices, including non-planar geometries. It has provision to include magnet misalignments. It does not treat longitudinal oscillations, although it does treat momentum oscillations. The program is therefore symplectic in four dimensions (transverse motion), but not in six (coupled longitudinal and transverse motion). This version of TEVLAT was used to study space charge effects in the Main Ring in a previous report [4]. Space charge effects were included using a "mean field" approximation, assuming a Gaussian beam in all planes. This means that only one test particle was tracked, and the space charge force on this particle was calculated by assuming that the beam as a whole had a Gaussian profile, centered on the closed orbit. This is a mean field approximation: a given particle responds to the average field of all the other particles. The standard deviations of the beam, in all the three planes, were not changed during the tracking. This approximation was also made when using TEVLAT in the present report. It is of course an approximation whose validity must be justified. To do so, a program was written to track a distribution of particles, and the results will be described below. The Laslett tuneshift [6], which is the space charge induced tuneshift of small amplitude particles, was successfully reproduced by the tracking [4]. Stable resonance islands, caused by the crossing of low order resonance lines by particles close to the beam core, were also observed. The results of the tracking studies of Ref. [4] were, however, inconclusive. No evidence was found that space charge leads to any observable difference in the particle motion. However, even without space charge, the particle motion did not display any easily quantifiable pattern. Hence it was difficult, if not impossible, to formulate a criterion to say that the results with and without space charge were the "same" (or "different").

It was also significant that synchrotron oscillations were not treated in Ref. [4], because particle loss induced by space charge is believed to be much worse when synchrotron oscillations are present. It is believed that particles are carried to regions of bad magnetic field quality by a combination of momentum offset and dispersion, and these "bad fields" drive unstable resonances in the particle motion. Coupled with the crossing of resonance lines induced by space charge, the particles are lost. In fact, particles can be lost merely by the action of the synchrotron oscillations alone, without the presence of space charge. Hence TEVLAT was modified to treat longitudinal motion, and to be fully symplectic in six dimensions. Thus the input to TEVLAT now requires the user to specify the r.m.s. bunch length (in nanoseconds), as well as the r.m.s. relative momentum spread.

In a parallel and independent development, L. Michelotti has written an interactive graphics program called AESOP [5], to display action-angle variables in four dimensional (transverse) phase space, by displaying suitable three dimensional sections of the particle orbits. The results are displayed on an Evans and Sutherland PS-390 computer/terminal, and can be manipulated (rotated, magnified) in real time. This enables one to observe tori, resonance islands, separatrices, etc., for resonances in two degrees of freedom. The user must supply the tracking data, e.g. by writing a subroutine to track a particle through a lattice. The subroutine is compiled and then linked to AESOP. TEVLAT has been interfaced to AESOP by changing it into a collection of subroutines. In this way, Main Ring tracking data can be displayed graphically, and manipulated to observe the four dimensional surfaces and resonances. Unfortunately, the results are useful only when seen on a screen. Hardcopy images reveal relatively little information. Therefore there are no graphs of TEVLAT output in this report.

In any case, the results are still inconclusive. The tracking, especially with space charge, is extremely slow. There is still no method to characterize the particle behavior, so as to *quantify any difference in behavior when space charge is included. To date, there is no* evidence that space charge causes significant particle loss, or causes any other systematic difference in the particle motion other than an amplitude-dependent tuneshift, negative and larger in magnitude for small amplitude particles. There are still serious fundamental questions concerning the basic tracking procedure, without the inclusion of space charge. The use of TEVLAT on different computers, but with the same model lattice and input parameters, yields different results. In general, the results are widely different when the dynamics is nonlinear. There is, as yet, no result for the dynamic aperture of the Main Ring, using particle tracking without space charge. This is partly because a precise definition for the dynamic aperture has not yet been formulated. However, Michelotti and coworkers are taking active steps to study particle tracking [7], as stated above.

# 4 Multiparticle Tracking

This report *will*, however, present results from a new multiparticle tracking program, called MULTI. *It is also a kick code, but fully symplectic in six dimensions. So far it has been* applied only to idealized lattice models. The models consist of FODO cells (exact linear dynamics), with one nonlinear multipole (sextupole or octupole) per cell, and one RF cavity in the ring. Hence it treats synchrotron oscillations and also the coupling to the path length of an orbit. It uses a mean field approximation, assuming a Gaussian beam in all planes, but the standard deviations (beam sizes) are updated at the end of each turn. This is done by calculating the standard deviations, in the various planes, of the distribution of test particles. Typically 100 to 1000 particles are tracked. The beam emittance is plotted as a function of time, or turn number. Sample results are shown in Figs. 2 – 8. Only the horizontal emittance is plotted. The contribution of the dispersion and momentum spread to the horizontal beam size has been subtracted. A commentary on the results follows.

1. Fig. 2: pure linear dynamics, i.e. tracking through a FODO lattice. There are no synchrotron oscillations. The graph shows the horizontal transverse emittance (95%, normalized) as a function of turn number. The emittance is constant, as expected.
2. Fig. 3: as in Fig. 2. The graph shows a Fourier transform of the small amplitude particle motion, and has a sharp line at the betatron tune, as expected.
3. Fig. 4: as in Fig. 3, but including space charge (but no nonlinear multipoles). There is again a sharp line, but at a lower tune. The tuneshift agrees with the Laslett formula [6]. There are $2 \times 10^{10}$ particles per bunch, and 1113 bunches. The initial emittance (95%, normalized) is $10\pi$ mm-mrad and the r.m.s. bunch length is 2 nsec, i.e. 0.6 m.
4. Fig. 5: tracking with space charge but no nonlinear multipoles, as in Fig. 4, but this time a graph of emittance vs. turn number. There does not appear to be any emittance growth.
5. Fig. 6: tracking with sextupoles but no space charge. The sextupole strength $s$ is defined via $\Delta x' = s(x^2 - y^2)$. Here $x$ and $y$ are measured in meters, and $x'$ is measured in radians. There does not appear to be any emittance growth.
6. Fig. 7: tracking with both space charge and sextupoles. The test distribution contains 400 particles. There is a slight emittance growth initially, then the graph flattens out.
7. Fig. 8: as in Fig. 7, but with a test distribution of 100 particles, but tracked for 10,000 turns. There is no evidence for emittance growth. This graph is reproduced from Ref. [9], and was magnified several times, hence the poor quality of the reproduction.

The multiparticle tracking used a model lattice with approximately the same circumference, tunes, beta functions, dispersion, RF voltage and synchrotron period, and beam emittances as in the Main Ring at 8 GeV. It obviously did not have the full set of nonlinear multipoles present in the Main Ring. With 100 or more particles in the test distribution, the emittance did not display any noticeable statistical fluctuations (distributions of 10 or 50 particles did). There is no evidence for emittance growth, or particle loss (growth of individual particle amplitudes to more than 50 mm — taken as the aperture size), even in the longest runs.

# 5 Conclusion

Numerous results on particle tracking were presented at a workshop on particle tracking held in Fermilab recently [8], and they all treated transverse motion only, and ignored many-body effects. It was admitted at this meeting that one does not yet know how to calculate the dynamic aperture of a circular accelerator, or to predict the long-term stability of a trajectory except in highly idealized cases. Nevertheless, I have attempted to track particles in six dimensions, and to include space charge, and to produce graphs comparable to those observed experimentally, as in Fig. 1. I have found no conclusive results whatsoever to date, either with or without space charge.

However, an active effort is being made by Michelotti and coworkers [7] to study particle tracking, and to build it up into a useful tool. Results have been presented in Refs. [5] and [8]. To date they have not treated many-body phenomena such as space charge, but it is to be hoped that they do so and succeed in elucidating its effects.

## ACKNOWLEDGEMENT

This work was supported by the Universities Research Association Inc., under Contract DE-AC02-76CH03000 from the Department of Energy.

## References

[1] D. Trbojevic (private communication). Similar graphs can be obtained using the Fast Time Plot (FTP) in the Main Control Room. Fig. 1 is a good example, showing clearly the reduction in beam intensity at low energy, a clear drop in intensity at transition, and negligible particle loss thereafter.

[2] The Main Ring lattice used here was obtained from R. Gerig (private communication). It includes both systematic and random magnet multipoles, and overpasses at both B0 and D0. Gerig also supplied a copy of the particle tracking program TEVLAT, which was subsequently modified as described in the text.

[3] TEVLAT was originally written by A. Russell, but has been modified by N. Gelfand and R. Gerig, and is now available on several Fermilab computers. Unfortunately the versions are not all identical; there is no "standard TEVLAT."

[4] S.R. Mane, "Space Charge Effects in the Main Ring at 8 GeV," TM-1510 (1988).

[5] L. Michelotti, "Exploratory Orbit Analysis," FNAL Conf 89-96 (1989), presented at the IEEE Particle Accelerator Conference, Chicago, March 20 - 23, 1989. The acronym AESOP stands for Analysis and Exploration of the Simulated Orbits of Particles.

[6] L. J. Laslett, "On Intensity Limitations Imposed by Transverse Space-Charge Effects in Circular Particle Accelerators," BNL 7534, (1963).

[7] This work is being carried out by FUDGE, the Fermilab Underground Dynamics Group Experiment, organized by L. Michelotti. Both analytical and numerical tools are being developed by the members, to study various model accelerator systems.

[8] A workshop on particle tracking and dynamic aperture studies, including results from the E778 experiment (FNAL Tevatron), the CERN Sp$\bar{p}$S dynamic aperture experiment, and the SSC/CDG, was held at Fermilab, March 16-17, 1989. The proceedings are unfortunately available only informally, as photocopies of transparencies.

[9] S.R. Mane, "Space Charge Effects in the Fermilab Main Ring at 8 GeV," FNAL Conf 89-77 (1989), presented at the IEEE Particle Accelerator Conference, Chicago, March 20 - 23, 1989.

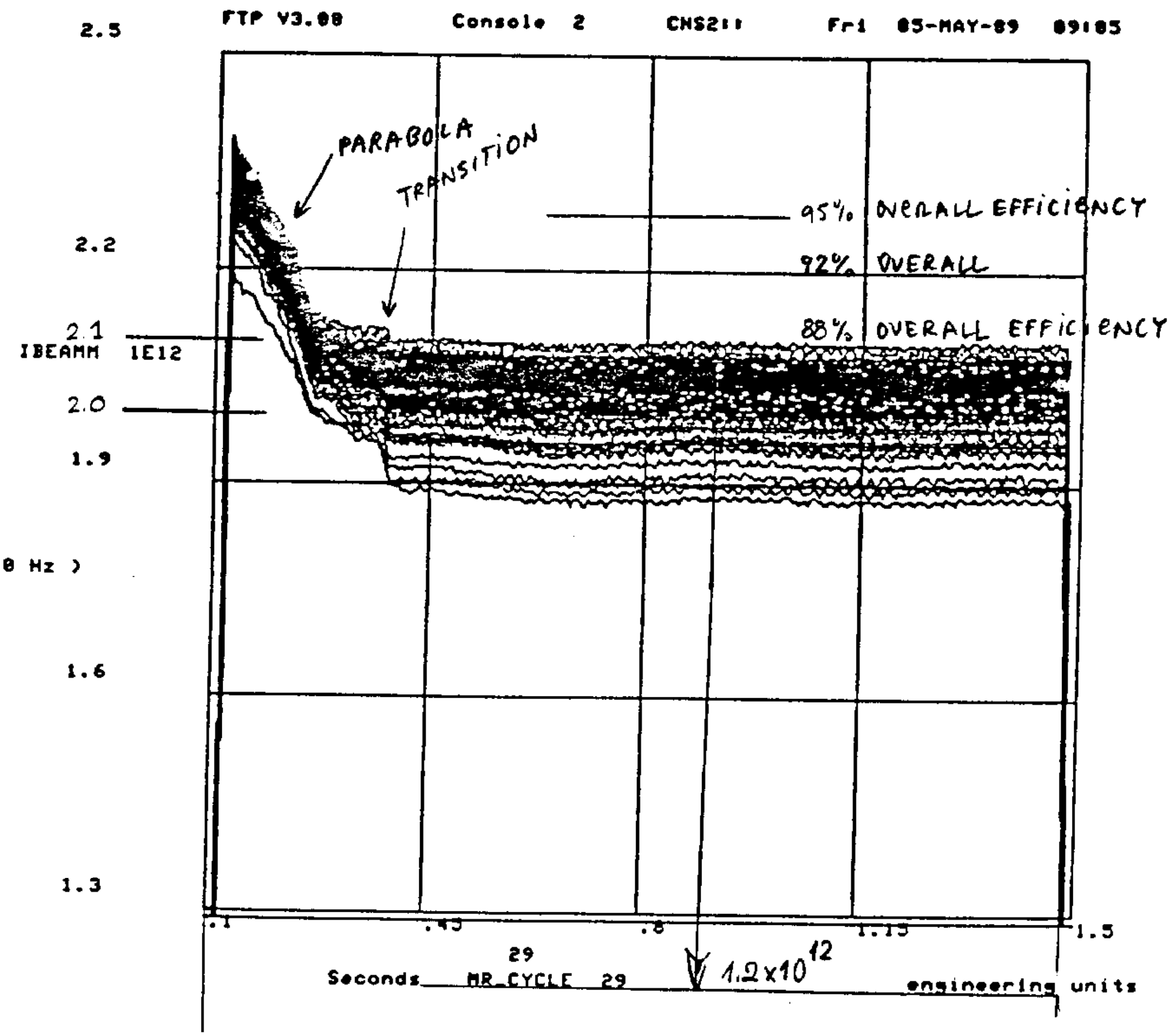


Fig. 1

Particle losses during the parabola and at transition are clearly visible.

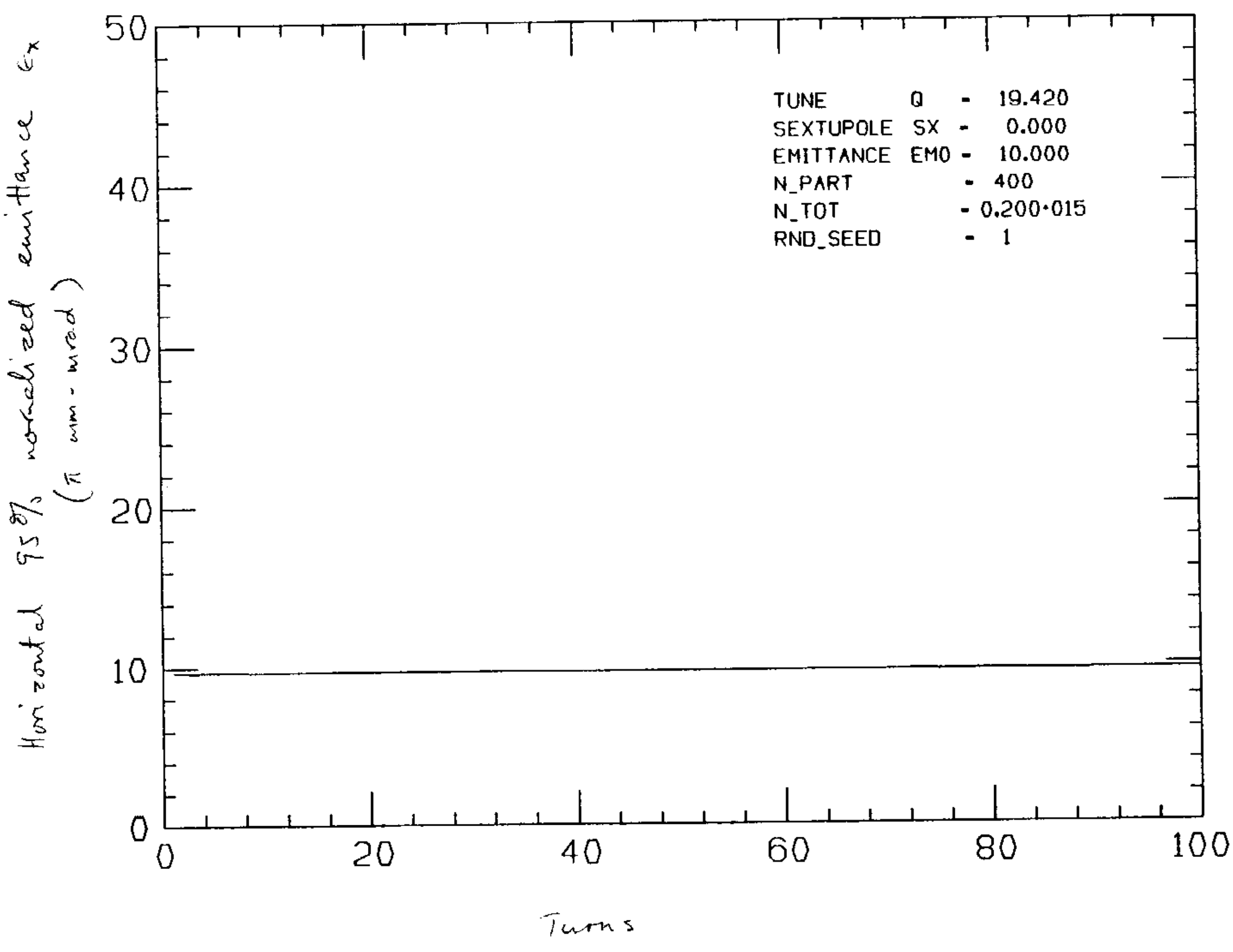


Fig. 2
The emittance is constant, as expected.

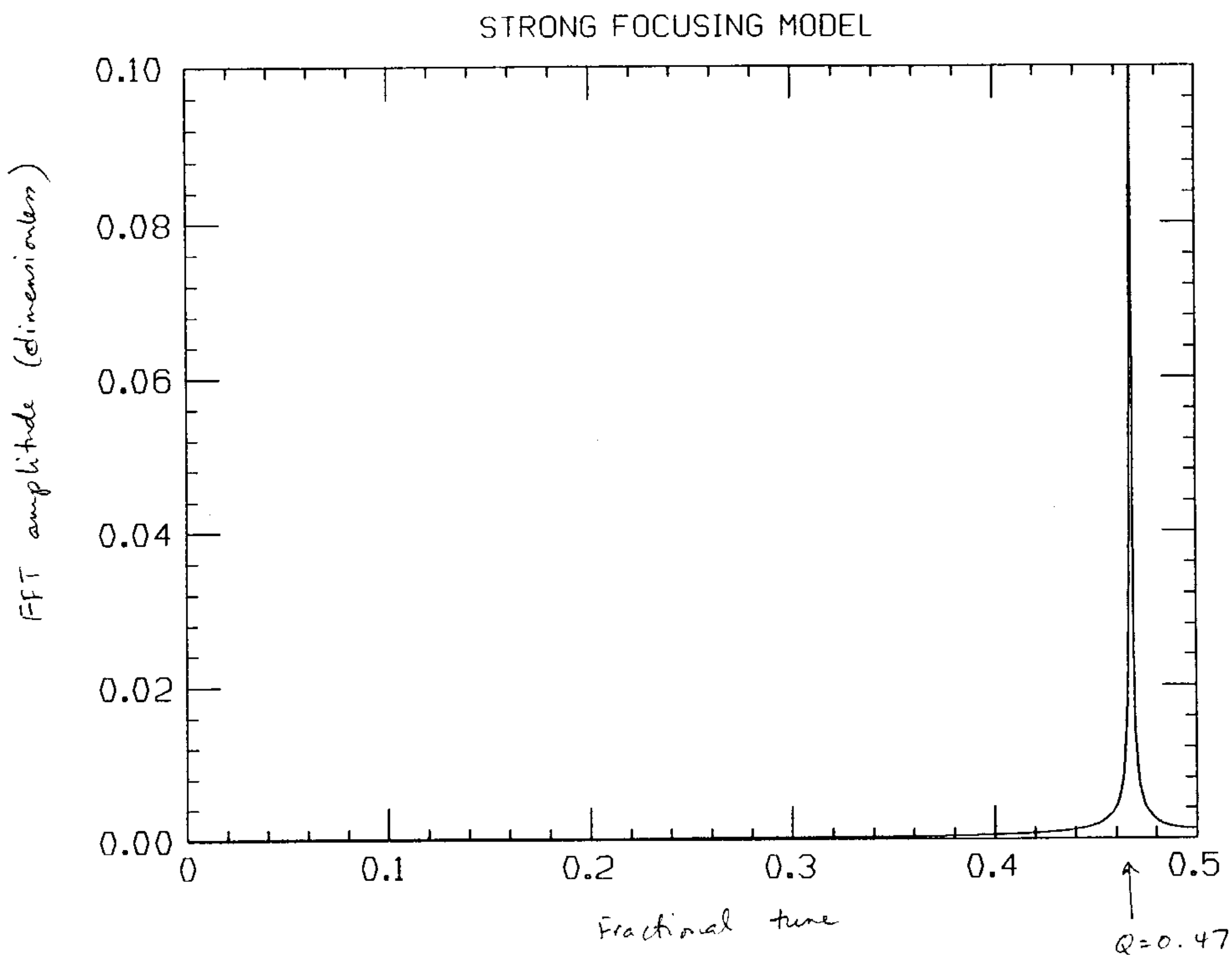


Fig. 3

The tune is 0.47, instead of 0.42 as demanded.
This is due to idealizations in the model.

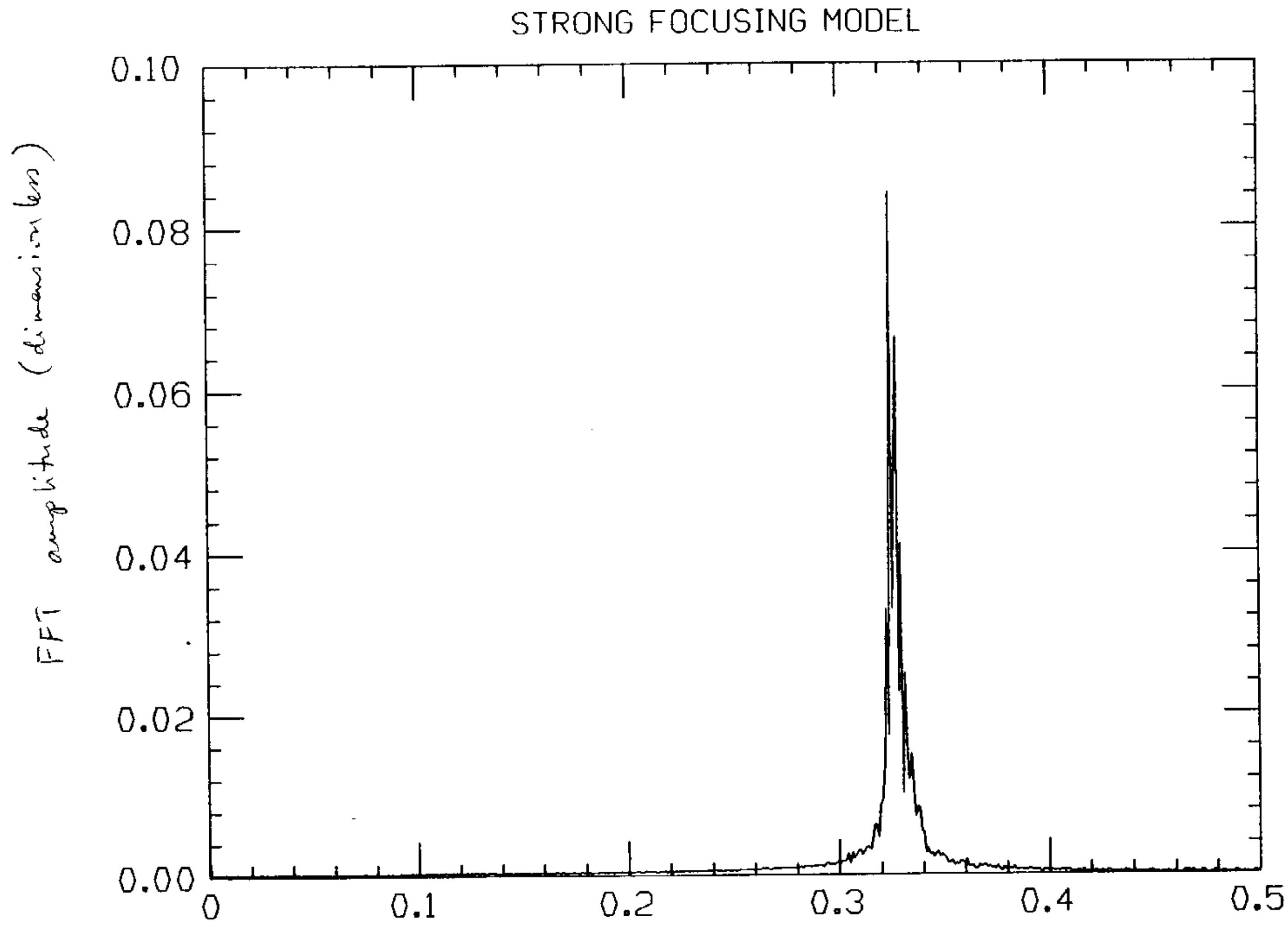


Fig 4
The small amplitude tune is reduced by the Laslett tune shift, as expected.

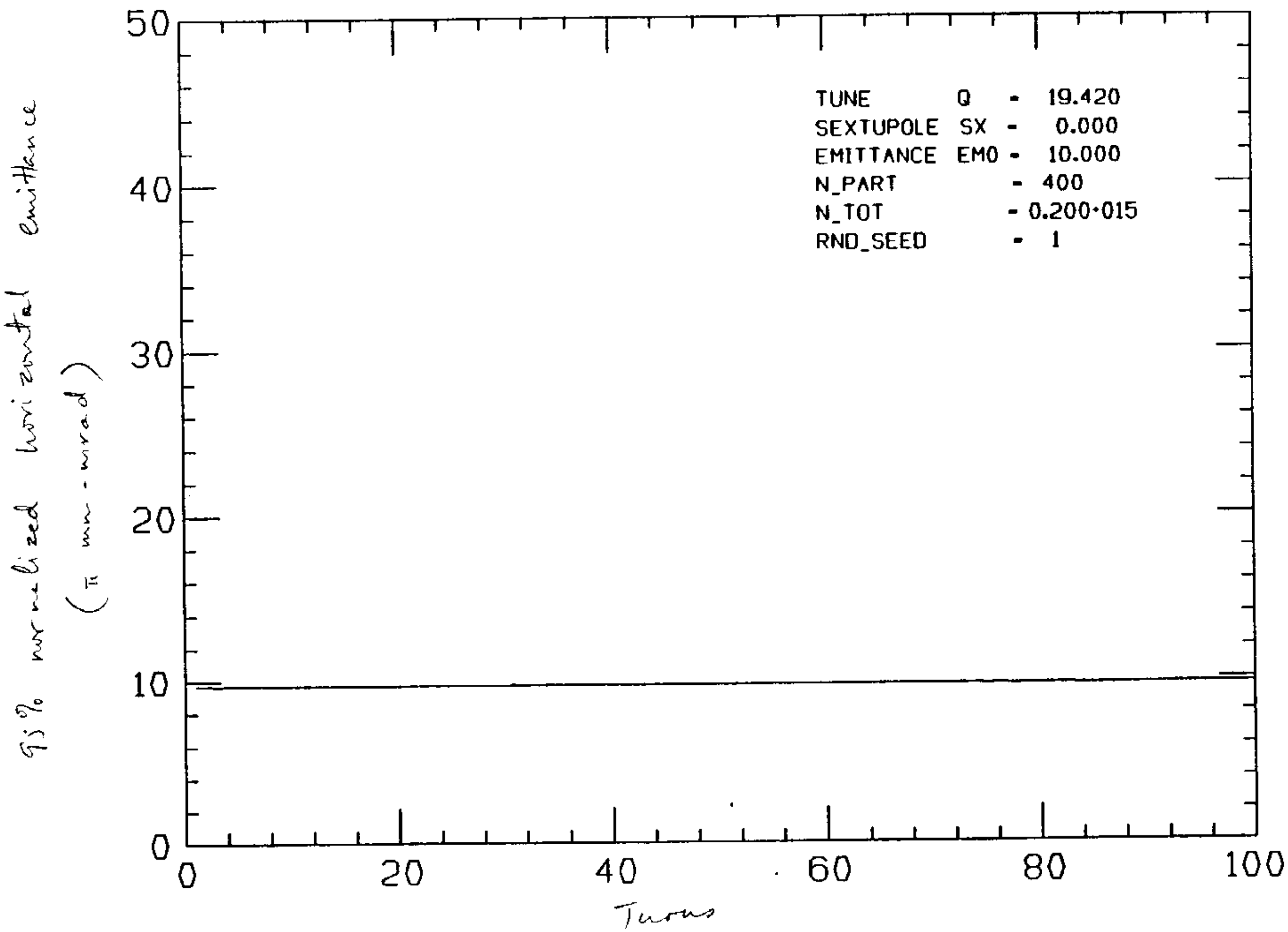


Fig 5

With space charge, but without sextupoles or synchrotron oscillations.

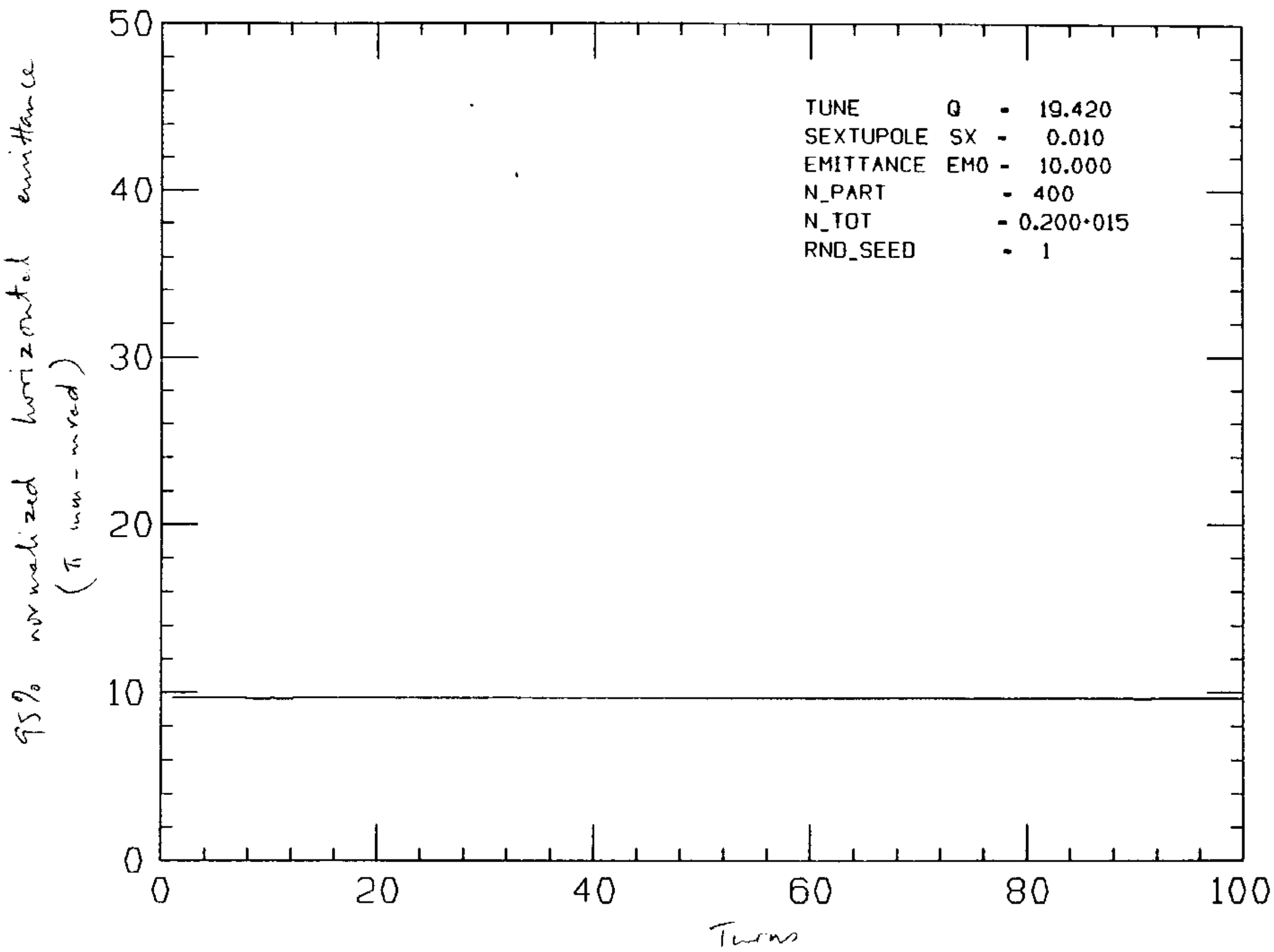


Fig 6.

With sextupoles but without space charge or synchrotron oscillations.

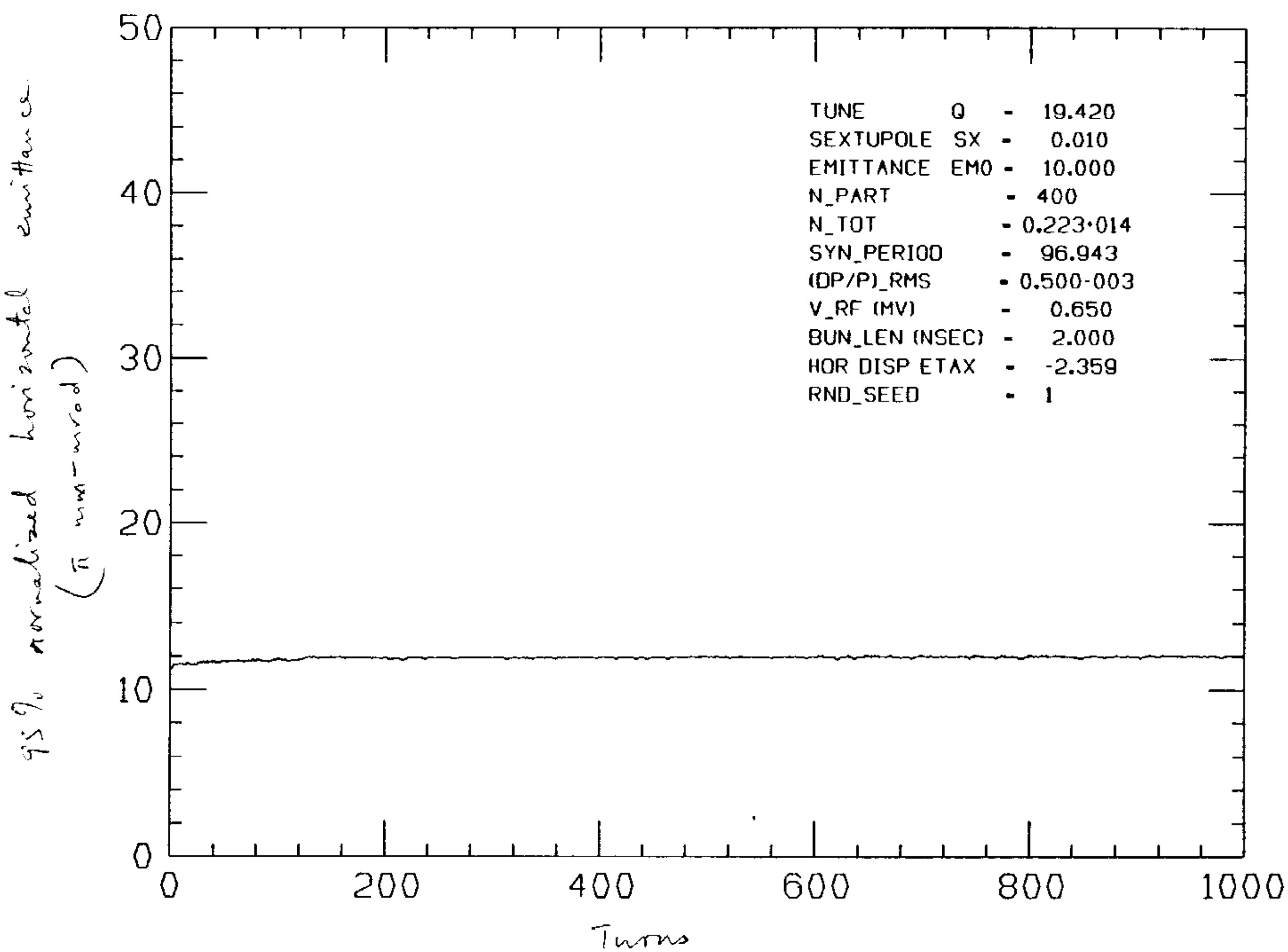
50
40
30
20
10
0
0
200
400
600
800
1000
TUNE Q - 19.420
SEXTUPOLE SX - 0.010
EMITTANCE EM0 - 10.000
N_PART - 400
N_TOT - 0.223·014
SYN_PERIOD - 96.943
(DP/P)_RMS - 0.500-003
V_RF (MV) - 0.650
BUN_LEN (NSEC) - 2.000
HOR DISP ETAX - -2.359
RND_SEED - 1
95 % normalized horizontal emittance
(π mm-mrad)
Turns

Fig. 7

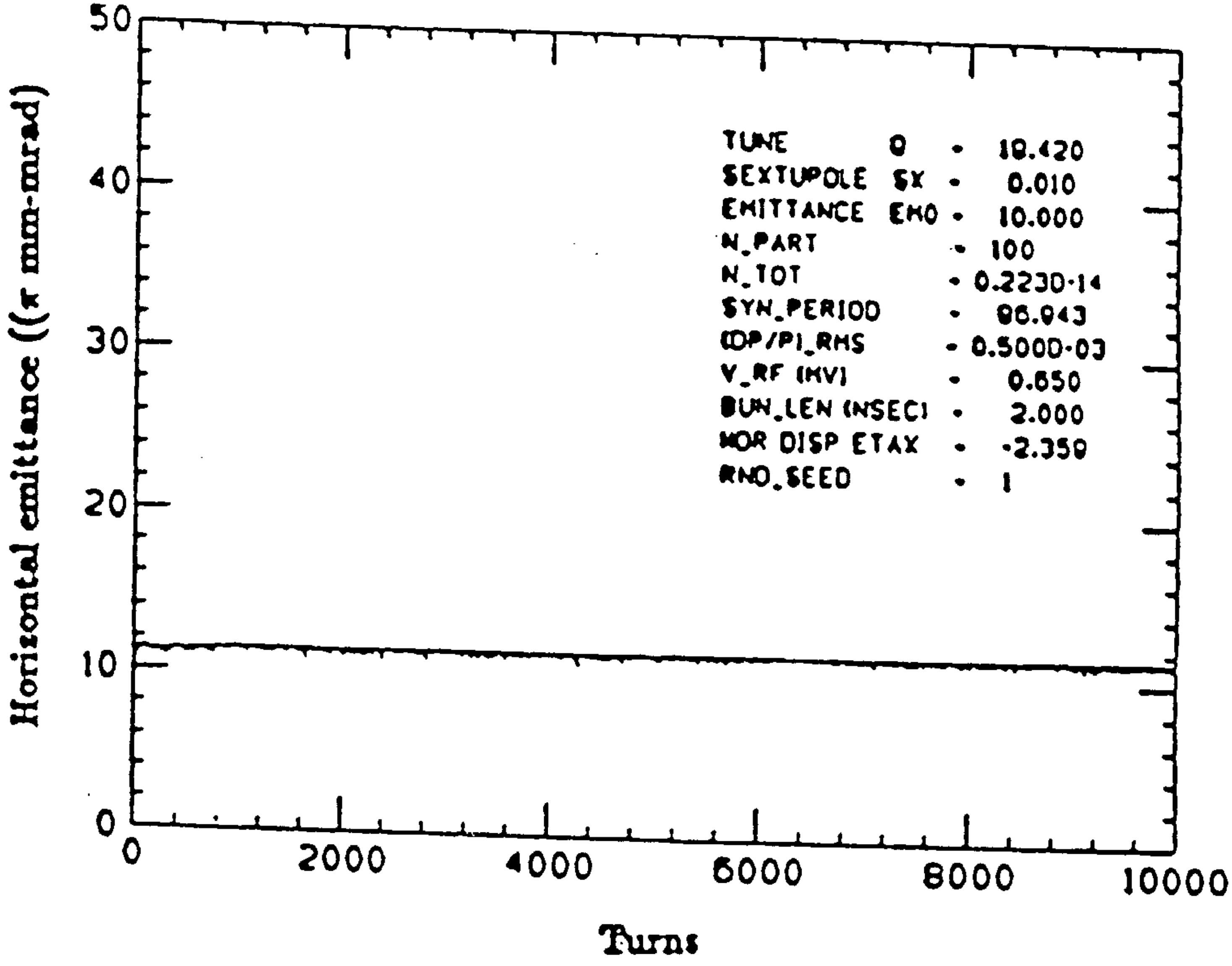
TUNE Q = 10.420
SEXTUPOLE SX = 0.010
EMITTANCE EMO = 10.000
N_PART = 100
N_TOT = 0.2230-14
SYN_PERIOD = 96.943
(DP/P)_RMS = 0.5000-03
V_RF (MV) = 0.650
BUN_LEN (NSEC) = 2.000
HOR DISP ETAX = -2.350
RND_SEED = 1
Horizontal emittance ((π mm-mrad)
Turns
0
10
20
30
40
50
2000
4000
6000
8000
10000

Fig. 8